

## Effective interactions in the delta-shells potential

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**Abstract** We determine two-body Skyrme force parameters from a Nucleon-Nucleon interaction as a function of the maximal momentum fitting NN scattering data. We find general agreement with  $V_{\text{lowk}}$  interactions based on high quality potentials.

**Keywords** Effective NN interactions · Skyrme forces

The use of effective interactions in Nuclear Physics is rather old and dates back to the pioneering works of Moshinsky [1] and Skyrme [2]. One of the advantages in doing so is that, as compared to *ab initio* calculations, the nuclear many body wave function has a much simpler structure since short range correlations play a marginal role allowing for a fruitful implementation of mean field Hartree-Fock calculations [3, 4, 5].

At the two body level the effective interaction of Moshinsky [1] and Skyrme [2] reads

$$\begin{aligned}
 V(\mathbf{p}', \mathbf{p}) &= \int d^3x e^{-i\mathbf{x} \cdot (\mathbf{p}' - \mathbf{p})} \hat{V}(\mathbf{x}) \\
 &= t_0(1 + x_0 P_\sigma) + \frac{t_1}{2}(1 + x_1 P_\sigma)(\mathbf{p}'^2 + \mathbf{p}^2) \\
 &\quad + t_2(1 + x_2 P_\sigma)\mathbf{p}' \cdot \mathbf{p} + 2iW_0 \mathbf{S} \cdot (\mathbf{p}' \wedge \mathbf{p}) \\
 &\quad + \frac{t_T}{2} \left[ \sigma_1 \cdot \mathbf{p} \sigma_2 \cdot \mathbf{p} + \sigma_1 \cdot \mathbf{p}' \sigma_2 \cdot \mathbf{p}' - \frac{1}{3} \sigma_1 \cdot \sigma_2 (\mathbf{p}'^2 + \mathbf{p}^2) \right] \\
 &\quad + \frac{t_U}{2} \left[ \sigma_1 \cdot \mathbf{p} \sigma_2 \cdot \mathbf{p}' + \sigma_1 \cdot \mathbf{p}' \sigma_2 \cdot \mathbf{p} - \frac{2}{3} \sigma_1 \cdot \sigma_2 \mathbf{p}' \cdot \mathbf{p} \right] + \mathcal{O}(p^4) \quad (1)
 \end{aligned}$$

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where  $P_\sigma = (1 + \sigma_1 \cdot \sigma_2)/2$  is the spin exchange operator with  $P_\sigma = -1$  for spin singlet  $S = 0$  and  $P_\sigma = 1$  for spin triplet  $S = 1$  states. In Ref. [6] the parameters of Eq. (1) were determined from just NN threshold properties such as scattering lengths, effective ranges and volumes without explicitly taking into account the finite range of the NN interaction.

Our aim here is to compute the Skyrme parameters from an analysis of NN scattering data below pion production threshold rather than to a fit to double-closed shell nuclei and nuclear matter saturation properties as it is usually done [3, 4, 5].

While the idea of effective interactions is conceptually simple and rather appealing computationally there is no unique or universal definition since any different method assumes a particular off-shell extrapolation which cannot be tested experimentally. On the other hand effective interactions such as Eq. (1) truly depend on the relevant wavelengths involved. This becomes clear in the  $V_{\text{lowk}}$  method [7, 8], where one truncates the model space Hamiltonian for states with CM momentum  $p \leq \Lambda$ . However, in order to be able to implement this method one needs a *choice* of a phenomenological potential which besides fitting the data satisfactorily provides the half-off-shell scattering amplitude.

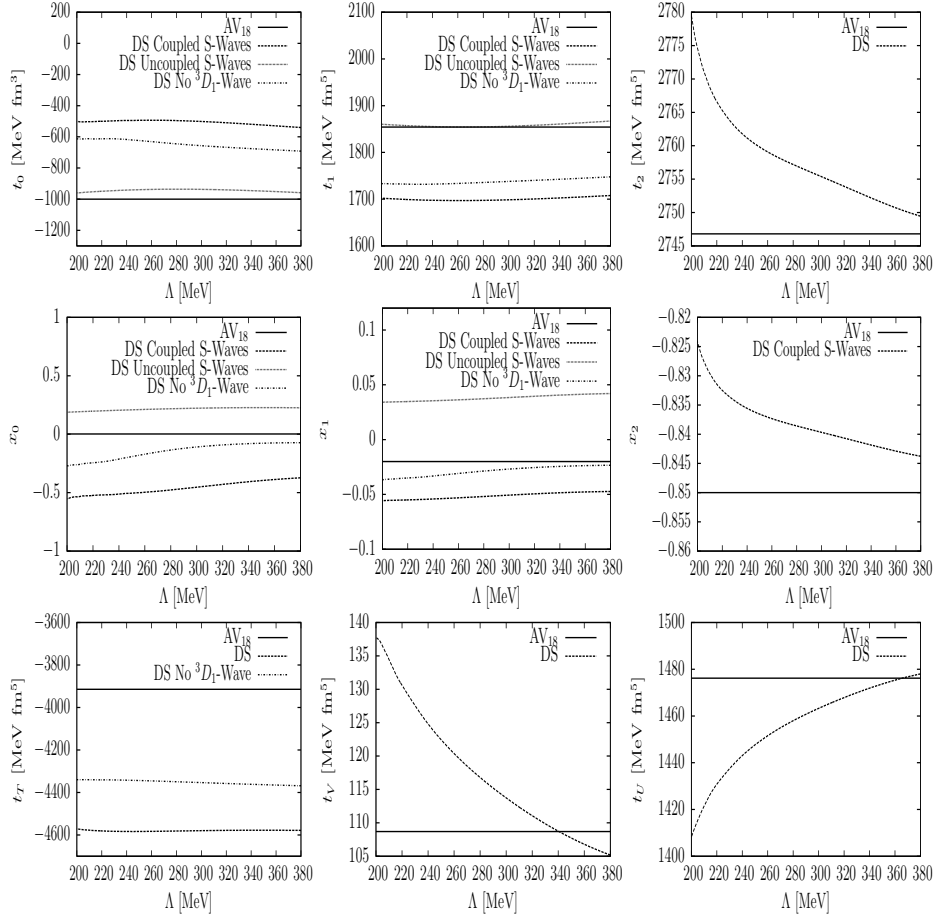
In the present contribution we define the interaction Eq. (1) by fitting a potential to NN phase-shifts below a given maximum CM momentum  $p \leq \Lambda$ . To proceed we use the delta-shells local potential for partial waves

$$V_{l,l'}^{JS}(r) = \sum_i (\lambda_{l,l'}^{J,S}) \delta(r - r_i) \quad r \leq 3\text{fm} \quad (2)$$

proposed by Avilés long ago [9] for the short range part. In addition we take the One-Pion-Exchange (OPE) and electromagnetic tail for  $r \geq 3\text{fm}$ . With this potential and below pion production threshold  $p \leq \sqrt{M_N m_\pi} = 362\text{MeV}$  a description of scattering observables can be achieved with  $\chi^2/\text{d.o.f} = 1.06$  [10], of comparable high quality as the bench-marking fits of the Nijmegen group [11, 12] and subsequent AV18 [13], CD-Bonn [14] and Spectator model [15] potentials. The rationale behind the schematic form of Eq. (2) is based on the expectation that a coarse graining of the interaction to a given wavelength should not display fluctuations of the interactions to shorter distances than  $\Delta r \sim 1/\sqrt{M_N m_\pi} = 0.54\text{fm}$ .

For the potential in Eq. (2) one finds after some calculation the result

$$\begin{aligned} (t_0, x_0 t_0) &= \frac{1}{2} \int d^3x [V_{3S_1}(r) \pm V_{1S_0}(r)] , \\ (t_1, x_1 t_1) &= -\frac{1}{12} \int d^3x r^2 [V_{3S_1}(r) \pm V_{1S_0}(r)] , \\ (t_2, x_2 t_2) &= \frac{1}{54} \int d^3x r^2 [V_{3P_0}(r) + 3V_{3P_1}(r) + 5V_{3P_2}(r) \pm 9V_{1P_1}(r)] , \\ t_V = W_0 &= \frac{1}{72} \int d^3x r^2 [2V_{3P_0}(r) + 3V_{3P_1}(r) - 5V_{3P_2}(r)] , \\ t_T &= \frac{1}{5\sqrt{2}} \int d^3x r^2 V_{E_1}(r) , \\ t_U &= \frac{1}{36} \int d^3x r^2 [-2V_{3P_0}(r) + 3V_{3P_1}(r) - V_{3P_2}(r)] , \end{aligned} \quad (3)$$



**Fig. 1** Dependence of the effective interaction parameters as a functions of the maximal CM momentum  $\Lambda$  (in MeV) for which the neutron-proton phase shifts have been fitted. We compare with the AV18- $V_{\text{lowk}}$  potential when  $\Lambda_{\text{lowk}} = 2.1\text{fm}^{-1}$

where the  $\pm$  in the first three equations refers to the first and second possibilities on the l.h.s. We use the delta-shells potential for  $r \leq r_c$ . The effective interaction due to OPE above  $r > r_c$  is given by the following formulas in the isospin invariant case

$$\begin{aligned}
 t_0|_{\text{OPE}} &= -\frac{f_{\pi NN}^2}{m_\pi^2} \Gamma(2, m_\pi r_c), & x_0 t_0|_{\text{OPE}} &= 0, \\
 t_1|_{\text{OPE}} &= \frac{f_{\pi NN}^2}{3m_\pi^4} \Gamma(4, m_\pi r_c), & x_1 t_1|_{\text{OPE}} &= 0, \\
 t_2|_{\text{OPE}} &= \frac{5f_{\pi NN}^2}{9m_\pi^4} \Gamma(4, m_\pi r_c), & x_2 t_2|_{\text{OPE}} &= -\frac{4f_{\pi NN}^2}{9m_\pi^4} \Gamma(4, m_\pi r_c), \\
 t_V|_{\text{OPE}} &= 0, \\
 t_U|_{\text{OPE}} &= \frac{2f_{\pi NN}^2}{15m_\pi^4} [3\Gamma(2, m_\pi r_c) + 3\Gamma(3, m_\pi r_c) + \Gamma(4, m_\pi r_c)],
 \end{aligned}$$

**Table 1** Skyrme parameters obtained from the AV18- $V_{\text{lowk}}$  with  $A_{\text{lowk}} = 2.1\text{fm}^{-1}$  and the charge dependent Delta-shells fit up to  $E_{\text{LAB}} = 350\text{MeV}$ . Both potentials reproduce the np phase-shifts up to pion production threshold.  $t_{0,1,2}$  are in  $(\text{MeVfm}^3)$ ,  $x_{0,1,2}$  are dimensionless and  $t_{U,V,W}$  are in  $(\text{MeVfm}^5)$ .

Parameter	$t_0$	$x_0$	$t_1$	$x_1$	$t_2$	$x_2$	$W_0$	$t_U$	$t_T$
$V_{\text{lowk}}$	-999.6	0.002	1854.2	-0.02	2198.3	-0.91	84.1	1235.2	-3864.0
Delta-shell	-555.3	-0.36	1711.8	-0.05	2746.8	-0.845	108.7	1476.2	-4576.4

$$t_T|_{\text{OPE}} = -\frac{2f_{\pi NN}^2}{5m_\pi^4} [3\Gamma(2, m_\pi r_c) + 3\Gamma(3, m_\pi r_c) + \Gamma(4, m_\pi r_c)], \quad (4)$$

where  $f_{\pi NN} = g_{\pi NN} m_\pi / 2M_N$ , with  $f_{\pi NN}^2 / (4\pi) \sim 0.08$  and  $\Gamma(n, x) = \int_x^\infty dt e^{-t} t^{n-1}$ . These OPE contributions are numerically tiny for  $r_c = 3\text{fm}$ . From the fit in Ref. [10] we get the results of Table 1. They are compared with the extraction of Ref. [6].

The scale dependence on the fitted np scattering phase-shifts is presented in Fig. 1. Note that the set of Eqs. (3) involve only S- and P-waves as well as the SD-wave mixing but no D-waves. However, the tensor force requires a non-vanishing D-wave. Having this in mind we distinguish three different situations for the case of the triplet  $^3S_1$  wave due to the role played by the tensor force. The “coupled case” corresponds to make a fit to the  $^3S_1, ^3D_1, E_1$  phase-shifts, whereas the “uncoupled case” is obtained from a fit of the  $^1S_3$  potential from the  $^3S_1$  phase shift, without considering the coupling to the  $^3D_1$  channel. Another intermediate case corresponds to just fit  $^3S_1$  and  $E_1$  phases taking a vanishing  $^3D_1$  potential. As can be seen from Fig. 1 the uncoupled case resembles best the  $V_{\text{lowk}}$ -value. Generally, there is a close agreement (note the y-axis scales) with the  $V_{\text{lowk}}$  results as applied to the AV18 potential when a value of  $A_{\text{lowk}} = 2.1\text{fm}^{-1}$  is taken. A more complete analysis properly weighting the relative importance of  $D$ - vs  $P$ - and  $S$ -waves with inclusion of uncertainties [16, 17] will be presented elsewhere. In any case, the enhanced attraction confirms the binding features of doubled closed shell nuclei outlined in Ref. [18].

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